EXERCISE – IV HINTS & SOLUTIONS

Sol.1 ℓn (4 × 12 × 36 × 108 × .... upto n terms)

$= \ell n \{(4) \times (4.3) \times (4.3^2) \times (4 \times 3^3) \times .... \times (4 \times 3^{n-1})\}$

$= \ell n\ (4^n \times 3^{1+2+3+....+(n-1)})$

$= \ell n\ 4^n + \ell n(3)^{\frac{n(n-1)}{2}} = 2n\ell n2 + \frac{n(n-1)}{2}\ell n3$

Sol.2 $1, A_1, A_2, ..., A_n, 31$ & $\frac{A_7}{A_{n-1}} = \frac{5}{9}$

$d = \frac{31-1}{n+1} = \frac{30}{n+1}$

$\frac{A_7}{A_{n-1}} = \frac{1+7d}{1+(n-1)d} = \frac{1+\frac{30}{n+1}\times 7}{1+\frac{(n-1)30}{n+1}} = \frac{5}{9}$

$\Rightarrow \frac{n+211}{31n-29} = \frac{5}{9} \Rightarrow 9n + 1899 = 155\,n - 145$

$\Rightarrow 146\,n = 2044 \qquad \Rightarrow n = 14$

Sol.3 $\sum_{n=2,4,6}^{180} n\sin n^o = \frac{1}{90}$ [2 sin2º + 4 sin4º + 6 sin 6º + ... + 178 sin 178º + 0]

$= \frac{1}{90}$ [180 sin2º + 180 sin 4º + ... + 180º sin 88º + 90 sin 90º]

= 2 [sin2º + sin4º + sin6º + ... + sin 88º] + 1

$= 2\,\frac{2\sin\left(44.\frac{2^o}{2}\right)}{\sin\left(\frac{2^o}{2}\right)}\sin\left(\frac{2^o+88^o}{2}\right) + 1$

$= \frac{2\sin 44^o \sin 45^o}{\sin 1^o} + 1 = \frac{\cos 1^o - \cos 89^o}{\sin 1^o} + 1$

= cot 1º – 1 + 1 = cot 1º

Sol.4 $S = a + ar + ar^2 + ... + ar^{n-1} \Rightarrow S = \frac{a(1-r^n)}{1-r}$

$P = a^n\, r^{(1+2+...+(n-1))} \Rightarrow P = a^n r^{\frac{n(n-1)}{2}}$

$R = \frac{1}{a} + \frac{1}{ar} + \frac{1}{ar^2} + ... + \frac{1}{ar^{n-1}}$

$\Rightarrow R = \frac{1}{ar^{n-1}}(1 + r + r^2 + ... + r^{n-1})$

$\Rightarrow R = \frac{1}{a^2r^{n-1}}(a + ar + ar^2 + ... + ar^{n-1})$

$\Rightarrow R = \frac{1}{a^2r^{n-1}}S \qquad \Rightarrow \frac{R}{S} = \frac{1}{a^2r^{n-1}}$

$\Rightarrow \left(\frac{R}{S}\right)^n = \frac{1}{a^{2n}r^{(n-1)n}} \Rightarrow \left(\frac{R}{S}\right)^n = \frac{1}{\left(a^n r^{\frac{n(n-1)}{2}}\right)^2}$

$\Rightarrow \left(\frac{R}{S}\right)^n = \frac{1}{P^2} \qquad \Rightarrow P^2\left(\frac{R}{S}\right)^n = 1$

Sol.5 Let last three terms a – 6, a, a + 6

& first and last terms equal

$\Rightarrow$ four number are (a + 6), (a – 6), a , (a + 6)

first three terms in G.P. $(a-6)^2 = a(a+6)$

$a^2 - 12a + 36 = a^2 + 6a \Rightarrow 18a = 36 \Rightarrow a = 2$

Set of no. are {8, –4, 2, 8}

Sol.6 $S_n = 7 + 77 + 777 + ... +$ n terms

$S_n = \frac{7}{9}$ [9 + 99 + 999 + ... + n terms]

$S_n = \frac{7}{9}[(10-1)+(10^2-1)+(10^3-1) +.... +(10^n-1)]$

$= \frac{7}{9}[10 + 10^2 + 10^3 + ... + 10^n - n]$

$= \frac{7}{9}\left[\frac{10(10^n-1)}{9} - n\right] = \frac{7}{81}[10^{n+1} - 9n - 10]$

Sol.7 2, a, b, c, 18

a + b + c = 25 ......(i)

2a = 2 + b ......(ii)

$c^2 = 18b$ ......(iii)

From (i) & (ii)

$c = 27 - 3a \Rightarrow c^2 = 3^2 (9 - a)^2$

From (ii) & (iii)

$c^2 = 36 (a - 1)$

$9 (9 - a)^2 = 36 (a - 1)$

$\Rightarrow a^2 - 18a + 81 = 4a - 4 \Rightarrow a^2 - 22a + 85 = 0$

$\Rightarrow a = 17$, or $a = 5$

$a = 17$ reject $\{\because$ 2, a, b in A.P.$\}$

$b = a + (5 - 2) \Rightarrow b = 8$

b, c, 18 in G.P. $\Rightarrow c^2 = \sqrt{8.18} = \sqrt{16 \times 9} \Rightarrow c=12$

Sol.8 A, a, B in A.P. $\Rightarrow 2a = A + B$

A, p, q, B in G.P. $\Rightarrow p^2 = Aq$ & $q^2 = Bp$

$$2a = \frac{p^2}{q} + \frac{q^2}{p} \Rightarrow p^3 + q^3 = 2apq$$

Sol.9 $T_1 = a, T_2 = b, T_3 = a^2 \Rightarrow 2b = a + a^2$

& a, a^2, b in G.P. $\Rightarrow a^4 = ab$

$\Rightarrow a(a^3 - b) = 0 \Rightarrow a \neq 0 \quad \therefore \quad a^3 = b$

(i) $2b = a + a^2 \Rightarrow 2a^3 = a + a^2$

$\Rightarrow 2a^2 - a - 1 = 0$

$\Rightarrow (a - 1)(2a + 1) = 0$

$\because a = 1 \Rightarrow b = 1$ {not possible}

or $a = \frac{-1}{2}$ & $b = -\frac{1}{8}$

(ii) a, a^2, b in G.P.

$$\Rightarrow a, a^2, a^3 \Rightarrow S_\infty = \frac{\frac{-1}{2}}{1+\frac{1}{2}} = \frac{-1}{2} \times \frac{2}{3} = -\frac{1}{3}$$

(iii) a, a^3, a^2 A.P.

$$-\frac{1}{2}, -\frac{1}{8}, \frac{1}{4} \Rightarrow d = \frac{3}{8}$$

$$S_{40} = \frac{40}{2}[(-1) + 39 \times \frac{3}{8}] = 20\,[-1 + \frac{117}{8}]$$

$$= 20 \times \frac{109}{8} = \frac{545}{2}$$

Sol.10 $a + 9d = \frac{1}{21}$ & $a + 20d = \frac{1}{10}$

$$\Rightarrow d = \frac{1}{210} \text{ \& } a = \frac{1}{210}$$

$$\frac{1}{a + (209)d} = \frac{1}{\frac{1}{210} + \frac{209}{210}} = 1$$

Sol.11 $T_n = \frac{n}{1 + n^2 + n^4} = \frac{1}{2}\left\{\frac{2n}{(n^2 + n + 1)(n^2 - n + 1)}\right\}$

$$T_n = \frac{1}{2}\left\{\frac{1}{n^2 - n + 1} - \frac{1}{n^2 + n + 1}\right\}$$

$$S_n = \sum T_n = \frac{1}{2} - \frac{1}{2(n^2 + n + 1)}$$

$$\Rightarrow S_n = \frac{n(n+1)}{2(n^2 + n + 1)} \text{ \& } S_\infty = \frac{1}{2}$$

Sol.12 a, (a + d), ..., a + (n – 1) d in A.P.

$$\frac{1}{a}, \frac{1}{a + d}, \ldots, \frac{1}{a + (n-1)d} \text{ in H.P.}$$

Given $a = \frac{1}{a} \Rightarrow a^2 = 1$

& $a + (n - 1) d = \frac{1}{a + (n - 1)d}$

First $\Rightarrow a = \pm 1$ & Last $a + (n - 1) d = \pm 1$

If first term is –1 then last term is + 1

r^{th} term of A.P. from the begining = –1 + (r – 1) d

r^{th} term of H.P. from the end = $\frac{1}{1 + (r - 1)(-d)}$

multiply = $[-1 + (r - 1)d] \times \frac{1}{[1 + (r - 1)(-d)]} = -1$

which is independent from r

Sol.13 $S = 1 + 2\left(1 + \frac{1}{n}\right) + 3\left(1 + \frac{1}{n}\right)^2 + 4\left(1 + \frac{1}{n}\right)^3 + \ldots$

Let $r = 1 + \frac{1}{n} = \frac{n+1}{n}$

$S_n = 1 + 2r + 3r^2 + 4r^3 + \ldots + nr^{n-1}$

$r.S_n = \quad r + 2r^2 + 3r^3 + \ldots + (n - 1) r^{n-1} + nr^n$

– – – – – –

$S_n(1 - r) = (1 + r + r^2 + r^3 + \ldots + r^{n-1}) - nr^n$

$$S_n(1 - r) = \frac{r^n - 1}{r - 1} - nr^n$$

$$\Rightarrow S_n\left(1-\frac{n+1}{n}\right)=\frac{(n+1)^n-n^n}{n^n}\times\frac{n}{(n+1-n)}-\frac{n(n+1)^n}{n^n}$$

$$\Rightarrow S_n\left(\frac{-1}{n}\right)=\frac{(n+1)^n}{n^{n-1}}-n-\frac{(n+1)^n}{n^{n-1}}$$

$\Rightarrow$ $S_n = (-n) \times (-n) = n^2$

Sol.14 **(i)** $S = 1 + 5 + 13 + 29 + 61 + ... + T_n$

$S = 1 + 5 + 13 + 29 + ... + T_{n-1} + T_n$

$0 = 1 + 4 + 8 + 16 + 32 + ... + (n^{th} \text{ term}) - T_n$

$\Rightarrow$ $T_n = 1 + 4 + 8 + 16 + 32 + ... + (n^{th} \text{ term})$

$$T_n = 1 + \frac{4(2^{n-1}-1)}{2-1} = 1 + 4\,(2^{n-1} - 1)$$

$T_n = -3 + 2.2^n$

$S_n = \Sigma T_n = -3\Sigma 1 + 2\Sigma 2^n$

$$= -3n + 2\,\frac{(2^n-1)2}{2-1} = -3n + 2^{n+2} - 4$$

(ii) $S = 6 + 13 + 22 + 33 + ... + T_n$

$S = 6 + 13 + 22 + ... + T_{n-1} + T_n$

$0 = 6 + 7 + 9 + 11 + ... + n^{th} - T_n$

$T_n = 6 + (7 + 9 + 11 + ... + (n-1)^{th})$

$$= 6 + \frac{(n-1)}{2}[14 + (n-2)2]$$

$= 6 + (n-1)(n+5)$

$T_n = n^2 + 4n + 1$

$S_n = \Sigma T_n = \Sigma n^2 + 4\Sigma n + \Sigma 1$

$$= \frac{n(n+1)(2n+1)}{6} + \frac{4n(n+1)}{2} + n$$

$$= \frac{n(n+1)}{6}\,[2n + 13] + n$$

Sol.15 $A = \frac{a+b}{2}$, $G^2 = ab$, $H = \frac{2ab}{a+b}$

$G = A-15$ & $H = A-27$

$G^2 = AH$

$(A-15)^2 = A(A-27)$

$A^2 - 30A + 225 = A^2 - 27A$

$\Rightarrow$ $A = 75$ $G = 60$

$a + b = 150$, $ab = 3600$

$$a + \frac{3600}{a} = 150$$

$\Rightarrow a^2 - 150a + 3600 = 0 \Rightarrow (a-120)(a-30)=0$

no. are 120, 30

Sol.16 **(i)** $S = \frac{1}{1\cdot4\cdot7} + \frac{1}{4\cdot7\cdot10} + \frac{1}{7\cdot10\cdot13} + ...$

$$T_n = \frac{1}{(3n-2)(3n+1)(3n+4)}\times\frac{[(3n+4)-(3n-2)]}{6}$$

$$T_n = \frac{1}{6}\left(\frac{1}{(3n-2)(3n+1)}-\frac{1}{(3n+1)(3n+4)}\right)$$

$$\Sigma T_n = S_n = \frac{1}{24}-\frac{1}{6(3n+1)(3n+4)},\ S_\infty = \frac{1}{24}$$

(ii) $\Sigma T_n = \sum_1^n r(r+1)(r+2)(r+3)$

$$= \sum r(r+1)(r+2)(r+3)\left[\frac{(r+4)-(r-1)}{5}\right]$$

$\Sigma T_n = \frac{1}{5}\,\Sigma[r(r+1)(r+2)(r+3)(r+4)$

$- (r-1)\,r\,(r+1)\,(r+2)\,(r+3))$

$\Sigma T_n = S_n = \frac{1}{5}\,(n(n+1)(n+2)(n+3)(n+4)]$

S_∞ = not defined

(iii) $\sum_{r=1}^{n} T_n = \sum \frac{1}{4r^2-1} = \sum\frac{1}{(2r-1)(2r+1)}$

$$= \frac{1}{2}\sum_{r=1}^{n}\left[\frac{(2r+1)-(2r-1)}{(2r-1)(2r+1)}\right]$$

$$= \frac{1}{2}\sum_{r=1}^{n}\left[\frac{1}{2r-1}-\frac{1}{2r+1}\right]$$

$S_n = \frac{1}{2}-\frac{1}{2(2n+1)}$ $\Rightarrow S_n = \frac{n}{2n+1}$ & $S_\infty = \frac{1}{2}$

(iv) $S = \frac{2}{2}\left[\frac{1}{4}+\frac{1.3}{4.6}+\frac{1.3.5}{4.6.8}+....\right]$

$$T_n = 2\left[\frac{1.3.5.....(2n-1)}{2.4.6.8....(2n+2)}\times[(2n+2)-(2n+1)]\right.$$

$$T_n = 2\left[\frac{1.3.5....(2n-1)}{2.4.6.8...2n} - \frac{1.3.5...(2n+1)}{2.4.6.8....(2n+2)}\right]$$

$$\sum T_n = S_n = 2\left[\frac{1}{2} - \frac{1.3.5.(2n+1)}{2.4\ 6\ 8(2n+2)}\right] S_\infty = \frac{2}{2} \Rightarrow S_\infty = 1$$

Sol.17 $S = \frac{1^2}{6} + \frac{2^2}{6^2} + \frac{3^2}{6^3} + ... + \frac{n^2}{6^n} + ... \infty$

$$\frac{S}{6} = \frac{1^2}{6^2} + \frac{2^2}{6^3} + ... + \frac{(n-1)^2}{6^n} + \frac{n^2}{6^{n+1}} ....$$

$$\frac{5S}{6} = \frac{1}{6} + \frac{(2^2-1^2)}{6^2} + \frac{(3^2-2^2)}{6^3} + ... + \frac{n^2-(n-1)^2}{6^n} + ...$$

$$\frac{5}{6}S = \frac{1}{6} + \frac{3}{6^2} + \frac{5}{6^3} + \frac{7}{6^4} + ...$$

$$\Rightarrow S = \frac{1}{5}\left[1 + \frac{3}{6} + \frac{5}{6^2} + \frac{7}{6^3} + ...\right]$$

$$\Rightarrow \frac{S}{6} = \frac{1}{5}\left[\frac{1}{6} + \frac{3}{6^2} + \frac{5}{6^3} + ...\right]$$

$$\Rightarrow \frac{5S}{6} = \frac{1}{5}\left[1 + \frac{2}{6} + \frac{2}{6^2} + \frac{2}{6^3} + ...\right]$$

$$\Rightarrow S = \frac{6}{25}\left[1 + 2\left(\frac{\frac{1}{6}}{1-\frac{1}{6}}\right)\right]$$

$$\Rightarrow S = \frac{6}{25}\left[1 + \frac{2}{5}\right] = \frac{6}{25} \times \frac{7}{5} = \frac{42}{125}$$

Sol.18 $T_r = \sqrt{1 + \frac{1}{r^2} + \frac{1}{(r+1)^2}}$

$$= \sqrt{\frac{(r^4+r^2+1) + 2r(r^2+r+1)}{r^2(r+1)^2}}$$

$$= \sqrt{\frac{(r^2+r+1)[r^2-r+1+2r]}{r^2(r+1)^2}} = \sqrt{\frac{(r^2+r+1)^2}{r^2(r+1)^2}}$$

$$T_r = \frac{r^2+r+1}{r(r+1)} = \frac{(r+1)^2 - r}{r(r+1)}$$

$$T_r = \frac{r+1}{r} - \frac{1}{r+1} \Rightarrow T_r = 1 + \frac{1}{r} - \frac{1}{r+1}$$

$$\sum_{r=1}^{1999} T_r = 1999 + 1 - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + ... + \frac{1}{1999} - \frac{1}{2000}$$

$$= 2000 - \frac{1}{2000} = n - \frac{1}{n} = n = 2000$$

Sol.19 L.H.S.$=(a_1^2 - a_2^2) + (a_3^2 - a_4^2) + ... + (a^2_{2k-1} - a^2_{2k})$

$= (a_1 - a_2)(a_1 + a_2) + (a_3 - a_4)(a_3 + a_4) + ...$
$+ (a_{2k-1} - a_{2k})(a_{2k-1} + a_{2k})$

$= -d[a_1 + a_2 + a_3 + a_4 + ... + a_{2k-1} + a_{2k}]$

$$= -d \times \frac{2k}{2}[a_1 + a_{2k}]$$

$$= \frac{(a_1 - a_{2k})k}{(2k-1)}(a_1 + a_{2k})$$

$$\therefore \{a_{2k} = a_1 + (2k-1)d \ \& \ d = \frac{a_{2k} - a}{(2k-1)}\}$$

$$= \frac{k}{(2k-1)}(a_1^2 - a^2_{2k}) = \text{R.H.S.}$$

Sol.20 $2x^3 - 19x^2 + 57x - 54 = 0$ — roots a/r, a, ar

$$\frac{a}{r}.\ a.\ ar = \frac{54}{2}$$

$\Rightarrow a^3 = 27 \qquad \Rightarrow a = 3$

$$\frac{3}{r} + 3 + 3.r = \frac{19}{2}$$

$\Rightarrow 6 + 6r + 6r^2 = 19r \quad \Rightarrow 6r^2 - 13r + 6 = 0$
$\Rightarrow (2r-3)(3r-2) = 0 \quad \Rightarrow (2r-3)(3r-2) = 0$

$$\Rightarrow r = \frac{3}{2}, \text{ or } r = \frac{2}{3}$$

for sum of infinite $|r| < 1 \quad \therefore r = 2/3$

$$S_\infty = \frac{(a/r)}{1-r} = \frac{3/(2/3)}{1-\frac{2}{3}} = \frac{9}{2} \times 3 = \frac{27}{2}$$

Sol.21 $10x^3 - cx^2 - 54x - 27 = 0$ — α, β, γ

$\Sigma\alpha = \frac{c}{10}$, $\Sigma\alpha\beta = -\frac{54}{10}$, $\alpha\beta\gamma = \frac{27}{10}$

& $\beta = \frac{2\gamma\alpha}{\alpha+\gamma} \Rightarrow \alpha\beta + \beta\gamma + \gamma\alpha = 3\gamma\alpha$

$\Rightarrow -\frac{54}{10} = 3\gamma\alpha \Rightarrow \gamma\alpha = -\frac{9}{5}$, $\beta = \frac{27}{10} \times \frac{5}{-9} \Rightarrow \beta = \frac{-3}{2}$

$(\alpha + \gamma) = \frac{2\alpha\gamma}{\beta} \Rightarrow \alpha + \gamma = 2 \times \frac{(-9)}{5} \frac{2}{(-3)}$

$\Rightarrow \alpha - \frac{9}{5\alpha} = \frac{12}{5} \Rightarrow 5\alpha^2 - 9 = 12\alpha$

$\Rightarrow (\alpha - 3)(5\alpha + 3) = 0 \Rightarrow \alpha = 3, \alpha = -\frac{3}{5}$

if $\alpha = 3$ $\gamma = -\frac{3}{5}$ $\beta = -\frac{3}{2}$

$10.3^3 - c.3^2 - 54.3 - 27 = 0 \Rightarrow c = 9$

Sol.22 a, b, c, d, e

$2b = a + c$ & $c^2 = bd$ & $d = \frac{2ce}{c+e}$

(i) $\Rightarrow \frac{c^2}{b} = \frac{2ce}{c+e} \Rightarrow \frac{c.2}{(a+c)} = \frac{2e}{(c+e)}$

$\Rightarrow c^2 = ae \Rightarrow$ a, c, e in G.P.

(ii) $c = 2b - a$ & $ae = c^2 \Rightarrow e = \frac{c^2}{a} \Rightarrow \frac{(2b-a)^2}{a}$

(iii) $c = \sqrt{ae}$, $c = \sqrt{2.18} \Rightarrow c = 6$

$b = \frac{a+c}{2} \Rightarrow b = \frac{2+6}{2} \Rightarrow b = 4$

$d = \frac{2.6.18}{6+18} \Rightarrow d = \frac{18}{2} \Rightarrow d = 9$

Sol.23 $n^2(1 - ac) - n(a^2 + c^2) - (1 + ac) = 0$
$\Rightarrow (n^2 - 1) = (na + c)(nc + a)$
$\Rightarrow$ a, $H_1, H_2, \ldots, H_n$, c in H.P.

$\frac{1}{a}, \frac{1}{H_1}, \frac{1}{H_2}, \ldots, \frac{1}{H_n}, \frac{1}{c}$ in A.P.

common diff. = $d = \frac{(a-c)}{ac(n+1)}$

$\frac{1}{H_1} = \frac{1}{a} + d = \frac{1}{a} + \frac{a-c}{ac(n+1)} \Rightarrow H_1 = \frac{ac(n+1)}{cn+a}$

& $\frac{1}{H_n} = \frac{1}{a} + nd = \frac{1}{a} + \frac{n(a-c)}{ac(n+1)} \Rightarrow H_n = \frac{ac(n+1)}{(an+c)}$

$H_1 - H_n = ac(n+1)\left[\frac{1}{nc+a} - \frac{1}{na+c}\right]$

$= \frac{ac(n+1)(n-1)(a-c)}{(nc+a)(na+c)} = \frac{ac(a-c)(n^2-1)}{(nc+a)(na+c)} = ac(a-c)$

Sol.24 **(i)** $x + y + z = 15$ a, x, y, z, b in A.P.

$x + y + z = 3\left(\frac{a+b}{2}\right) = 15$, $\Sigma A_i = nA$

$a + b = 10$

$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{5}{3}$ & $\frac{1}{a}, \frac{1}{x}, \frac{1}{y}, \frac{1}{z}, \frac{1}{b}$ in A.P.

$\Rightarrow \frac{3(a+b)}{2ab} = \frac{5}{3}$, $\Sigma A_i = 3\left(\frac{\frac{1}{a}+\frac{1}{b}}{2}\right)$

$\Rightarrow 9(a + b) = 10ab$
$\Rightarrow ab = 9 \Rightarrow a = 1, b = 9$ or $b = 1, a = 9$

(ii) $xyz = \frac{15}{2}$ If a, x, y, z, b in A.P.

& $xyz = \frac{18}{5}$ if a, x, y, z, b in H.P.

a, x, y, z, b in A.P. $\Rightarrow d = \frac{b-a}{n+1} \Rightarrow d = \frac{b-a}{4}$

$x.y.z = \left(a + \frac{b-a}{4}\right)\left(a + \frac{2(b-a)}{4}\right)\left(a + \frac{3(b-a)}{4}\right)$

$x.y.z = \frac{(3a+b)(2a+2b)(a+3b)}{64}$

$\frac{(3a+b)(2a+2b)(a+3b)}{64} = \frac{15}{2}$ ... (i)

& $\frac{1}{a}, \frac{1}{x}, \frac{1}{y}, \frac{1}{z}, \frac{1}{b}$ in A.P., $d = \frac{(a-b)}{4ab}$

$$\frac{1}{x}\frac{1}{y}\frac{1}{z}=\left(\frac{1}{a}+\frac{(a-b)}{4ab}\right)\left(\frac{1}{a}+\frac{2(a-b)}{4ab}\right)\left(\frac{1}{a}+\frac{3(a-b)}{4ab}\right)$$

$$\frac{1}{x.y.z} = \frac{(3a+b)(2a+2b)(a+3b)}{64a^3b^3} = \frac{5}{18}$$

$$\Rightarrow \frac{15}{2a^3b^3} = \frac{5}{18} \quad \Rightarrow a^3b^3 = 27 \quad \Rightarrow ab = 3$$

a & b are positive integer.

$\Rightarrow$ a = 1 & b = 3 or a = 3 & b = 1

Sol.25 $S = \frac{1\cdot3}{2} + \frac{3\cdot5}{2^2} + \frac{5\cdot7}{2^3} + \frac{7\cdot9}{2^4} + \ldots \infty$

$$\frac{S}{2} = \frac{1.3}{2^2} + \frac{3.5}{2^3} + \frac{5.7}{2^4} + \ldots \infty$$

$-\quad -\quad -\quad -$

$$\Rightarrow \frac{S}{2} = \frac{1.3}{2} + 4\left[\frac{3}{2^2}+\frac{5}{2^3}+\frac{7}{2^4}+\frac{9}{2^5}\ldots\ldots\right]$$

$$S = 3 + 8\left[\frac{3}{2^2}+\frac{5}{2^3}+\frac{7}{2^4}+\frac{9}{2^5}+\ldots\right]$$

$$\frac{S}{2} = \frac{3}{2} + \qquad 8\left[\frac{3}{2^3}+\frac{5}{2^4}+\frac{7}{2^5}+\ldots\right]$$

$-\quad -$

$$\Rightarrow \frac{S}{2} = \frac{3}{2} + 8\left[\frac{3}{2^2}+\left(\frac{1}{2^2}+\frac{1}{2^3}+\frac{1}{2^4}+\ldots\right)\right]$$

$$\Rightarrow \frac{S}{2} = \frac{3}{2} + 8\left[\frac{3}{2^2}+\frac{\frac{1}{2^2}}{1-\frac{1}{2}}\right]$$

$$\Rightarrow S = 3 + 16\left[\frac{3}{4}+\frac{1}{2}\right] = 23$$

Sol.26 $x^3 + px^2 + qx - r = 0$ $0\begin{cases} a-d \\ a \\ a+d \end{cases}$

$(a + d) + a + (a - d) = -p \quad \Rightarrow \quad 3a = -p$

$\Rightarrow a = \frac{-p}{3}$ satisfy given equation $\frac{-p^3}{27}+\frac{p^3}{9}-\frac{pq}{3}-r=0$

$\Rightarrow 2p^3 - 9pq - 27r = 0$

& roots of other equation will be also in A.P.

$3a = 12 \Rightarrow a = 4$

$a(a^2 - d^2) = 28 \Rightarrow 16 - d^2 = 7 \Rightarrow d = \pm 3$

roots are $4 - 3, 4, 4 + 3 \Rightarrow 1, 4, 7$

or $4 + 3, 4, 4 - 3 \Rightarrow 7, 4, 1$

Sol.27 a, b, c in G.P. $\Rightarrow b^2 = ac$

common diff. $d = \log_b c - \log_c a = \log_a b - \log_b c$

$$\Rightarrow \log_b c - \log_c \frac{b^2}{c} = \frac{1}{\log_b \frac{b^2}{c}} - \log_b c$$

$$\Rightarrow \log_b c - 2\log_c b + 1 = \frac{1}{2-\log_b c} - \log_b c$$

Let $t = \log_b c$

$$d = t - \frac{2}{t} + 1 = \frac{1}{2-t} - t \Rightarrow \frac{t^2+t-2}{t} = \frac{t^2-2t+1}{(2-t)}$$

$$\Rightarrow \frac{(t+2)(t-1)}{t} = \frac{(t-1)^2}{(2-t)} \ \{\because t-1 \neq 0\}$$

$$\Rightarrow 4 - t^2 = t^2 - t \Rightarrow 2t^2 - 4 = t \Rightarrow 2(t^2 - 2) = t$$

$$\Rightarrow 2\left(t-\frac{2}{t}\right) = 1 \Rightarrow \left(t-\frac{2}{t}\right) = \frac{1}{2}$$

$$\therefore d = \left(t-\frac{2}{t}\right) + 1 = \frac{1}{2} + 1 = \frac{3}{2}$$

Sol.28 $G_1 \Rightarrow a + ar + ar^2 + \ldots. \infty = 1$

$G_2 \Rightarrow A + AR + AR^2 + \ldots \infty = 1$

$$1 = \frac{a}{1-r} = \frac{A}{1-R} \ \& \ ar = AR \Rightarrow \frac{a}{A} = \frac{R}{r}$$

$$\Rightarrow \frac{a}{A} = \frac{1-r}{1-R} = \frac{R}{r} \Rightarrow r - r^2 = R - R^2$$

$$\Rightarrow r - R = r^2 - R^2 \Rightarrow 1 = r + R \Rightarrow R = 1 - r$$

$$1 = \frac{a}{1-r} \Rightarrow a = 1 - r$$

$ar = AR \Rightarrow A = r$

$G_1 : (1 - r) + (1 - r)\,r + (1 - r)\,r^2 + \ldots.$

$G_2 : r + r(1 - r) + r(1 - r)^2 + \ldots.$

given $(1 - r)\,r^2 = \frac{1}{8}$

$\Rightarrow 8r^3 - 8r^2 + 1 = 0$ satisfy $r = \frac{1}{2}$

$(2r - 1)(4r^2 - 2r - 1) = 0$

$r \neq \frac{1}{2}$ $\because$ Two distinct G.P.

$\therefore$ $4r^2 - 2r - 1 = 0 \Rightarrow r = \frac{2 \pm \sqrt{20}}{8} = \frac{1 \pm \sqrt{5}}{4}$

If $r = \frac{1+\sqrt{5}}{4}$ then second term

$$r(1-r) = \left(\frac{1+\sqrt{5}}{4}\right)\left(\frac{3-\sqrt{5}}{4}\right) = \frac{2\sqrt{5}-2}{16} = \frac{\sqrt{5}-1}{8}$$

If $r = \frac{1-\sqrt{5}}{4}$ then second term

$$r(1-r) = \left(\frac{1-\sqrt{5}}{4}\right)\left(\frac{3+\sqrt{5}}{4}\right) = \frac{-2\sqrt{5}-2}{16} = \frac{-\sqrt{5}-1}{8}$$

which in form $\frac{\sqrt{m}-n}{p}$ $\Rightarrow$ m = 5, n = 1, p = 8

100m + 10n + p = 500 + 10 + 8 = 518

Sol.29 $2000x^6 + \underbrace{100x^5 + 10x^3 + x}_{\text{G.P.}} - 2 = 0$

$$\Rightarrow 2000x^6 + \frac{x[(10x^2)^3 - 1]}{10x^2 - 1} - 2 = 0$$

$$\Rightarrow 2(1000x^6 - 1) + \frac{x[1000x^6 - 1]}{10x^2 - 1} = 0$$

$$\Rightarrow (1000x^6 - 1)\left[\frac{x}{10x^2 - 1} + 2\right] = 0$$

$\Rightarrow 10^3x^6 = 1$ or $\frac{x}{10x^2 - 1} = -2$

$x^2 = \frac{1}{10}$ (reject) or $20x^2 + x - 2 = 0$

$x = \frac{-1+\sqrt{161}}{40}$ or $\frac{-1-\sqrt{161}}{40}$

$\therefore$ m = – 1, n = 161, r = 40

m + n + r = 200

Sol.30 G.P. is $a, ar, ar^2, \ldots\ldots\ldots ar^{n-1}$

given $\frac{a_1 + a_2 + \ldots\ldots + a_{11}}{a_n + a_{n-1} + \ldots\ldots + a_{n-10}} = \frac{1}{8}$

$$\Rightarrow \frac{\frac{a(r^{11}-1)}{(r-1)}}{\frac{ar^{n-1}\left(\frac{1}{r^{11}} - 1\right)}{\left(\frac{1}{r} - 1\right)}} = \frac{1}{8} \Rightarrow r^{n-11} = 8$$

& $\frac{a_{10} + a_{11} + \ldots. + a_n}{a_1 + a_2 + \ldots. + a_{n-9}} = 2$

$$\Rightarrow \frac{ar^9(r^{n-9} - 1)}{(r-1)} \times \frac{(r-1)}{a(r^{n-9} - 1)} = 2$$

$\Rightarrow r^9 = 2$ $\Rightarrow r = 2^{1/9}$ $\therefore$ $(2^{1/9})^{n-11} = 8$

$\Rightarrow (2)^{\frac{n-11}{9}} = (2)^3$ $\Rightarrow \frac{n-11}{9} = 3$ $\Rightarrow$ n = 38

Sol.31 Let the no. be

$100x + 10y + z$ $\Rightarrow y^2 = xz$

given $100x + 10y + z - 792 = 100z + 10y + x$

$\Rightarrow 100(x - z) - (x - z) = 792$

$(x - z) = \frac{792}{99}$ $\Rightarrow x - z = 8$

Also given

$x - 4, y, z$ in A.P.

$2y = x + z - 4$

$\Rightarrow 4y^2 = (x + z - 4)^2$

$\Rightarrow 4xz = (x + z)^2 - 8(x + z) + 16$

$\Rightarrow (x + z)^2 - (x - z)^2 = (x + z)^2 - 8(x + z) + 16$

$\Rightarrow -64 = 8(x + z) + 16$

$\Rightarrow (x + z) = \frac{80}{8}$ $\Rightarrow x + z = 10$

$\left.\begin{matrix} x - z = 8 \\ x + z = 10 \end{matrix}\right\} \Rightarrow x = 9$ & $z = 1$

$y^2 = xz \Rightarrow y^2 = 9$ $\Rightarrow y = 3$

no. is 931